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DOI: 10.53704/fujnas.v2i2.29

A publication of College of Natural and Applied Sciences, Fountain University, Osogbo, Nigeria

Journal homepage: [www.fountainjournals.com](http://www.fountainjournals.com)

ISSN: 2354-337X (Online), 2350-1863 (Print)

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## The use of Gamma Distribution to Evaluate Water Pollutants in Asejire Reservoir, Ibadan

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### Abstract

This research work under-studies the application of probability functions to water pollution data with some related works. It figures out two parameter gamma distribution fitted to monitor the concentration of five water pollutants in Asejire reservoir; Turbidity, Colour, pH, Dissolved Oxygen and Alkalinity, by estimating the parameters and testing the goodness of fit. It was discovered that all the considered pollutants could be modeled with gamma distribution as confirmed by the Kolmogorov Smirnov (KS) test. It was also found that all the pollutants were significant except Dissolved Oxygen. The technique and results presented in this study provide a foundation for the use of gamma distribution to model water pollutants. The model is used as decision support tools for the management of the reservoir, as the attention of government and stake-holders must be drawn to keeping water safe for sustainable development.

**Keywords:** *Pollution, Kolmogorov Smirnov test, Gamma Distribution, Estimation, Shapiro-Wilk,*

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### Introduction

Many stake-holders believe that most health and socio-economic problems in Nigeria are caused by water pollution. It is therefore of paramount importance to give attention to water pollution monitoring in the country, considering the rate at which people are being seriously affected by polluted water. Asejire reservoir, one of the major sources of water supply to people of Ibadan, Oyo State Nigeria is perceived to be undergoing gradual pollution from human related activities at a rate that may constitute health and socio-economic problem to the people.

There are many probability distributions that could be successfully utilized to parameterize water pollutants as the critical component of these distributions is their flexibility to represent a variety of events. It should therefore be noted that there is little difference between many of the commonly used distributions when estimating parameters based on limited number of points, one of these available distributions is gamma, making it a good choice for application in this work.

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However, a lot of improvements and development have been recorded over decades as probability functions are used in environmental pollution monitoring (Gilbert *et al.*, 1982). He stated among other probability density functions, those that are extensively used as lognormal, Weibull, beta, gamma, and chi-square distribution. Liu *et al.* (2003) studied the application of water quality model in the white-cart water catchments, Glassglow, UK and received some good results in monitoring the quality of water. Donigian *et al.* (1981) also discussed the watershed hydraulic processes affecting water quality and offered an overview on the computer models developed. Zhang *et al.* (2004) researched to water quality models with computer. The scale and shape parameters that are common in probability distributions make them to take a variety of shapes and can be used to model both right and left skewed data sets. (Weber *et al.*, 1995) worked on statistical methods for assessing ground water compliance and clean-up. In his study, he recommended statistical procedures for making regulatory decisions to include hypothesis tests, statistical interval estimates, and trend analyses. Application of these methods within the context of regulatory ground-water monitoring programmes in USA was very much effective. Mu *et al.* (2004) established a hydrological statistical model for the river-basin of Jialu-River and Juweil River by applying multiple regression method. In the year 2001, Chen and Yaang, 2001 established an econometric model for urban water pollution loss. The need to ensure that all regulations or legislations clearly defined the analysis performance criteria to be met by laboratories carrying out environmental regulatory analysis of pollutants was discussed by (Thompson *et al.*, 2006). He focused on the importance of employing model fitting for the purpose of analysis to monitor pollution. Rice *et al.* (1995), Wackerly *et al.* (1996) described in some details the ideas of estimation and modeling. They both gave a very clear account of basic probability theory as well as good overview of statistical methods. Chandler *et al.* (2001) has also given a fantastic overview of

linear model in climate research. Moreso, (Gregory *et al.*, 2006) in their article "use of the gamma distribution to represent monthly rainfall in Africa for drought monitoring applications" demonstrated the feasibility of fitting cell-by-cell probability distributions to grids of monthly interpolated, continent-wide-data. In their study, it was discovered that the gamma distribution is suitable for roughly 98% of the locations over all months. The model was used as decision support tools for the management of water, agricultural resources and food reserves. Consequently, an understanding of what statistical method is appropriate given the type of data that have been collected is clearly important and also crucial to realize that different statistical methods have much in common as one method leads to another. In some cases, it may be necessary to fit different distributions for the data, in a way to see which one best fit by carrying out a goodness of fit test e.g. the Kolmogorov-Smirnov (Ks) test to unravel the true distribution of the data.

The primary objectives of this paper are to estimate and evaluate distribution parameters that may be used to describe the amount of pollutants in Asejire reservoir (Ibadan). More specifically probability distribution parameters are estimated from monthly data on water quality measures collected from the ministry of water resources, Oyo state, and the goodness of fit of the parameters are assessed.

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**Materials and Methods**

**Gamma Distribution**

In probability theory and statistics, the gamma distribution is a two-parameter family of continuous probability distributions. It has a scale and shape parameters, say  $\theta$  and  $K$  respectively. If  $K$  is an integer then the distribution represents the sum of  $K$  independent exponentially distributed random variables, each of which has a mean of  $\theta$  (which is equivalent to a rate parameter of  $\theta^{-1}$ ) Wackerly et al., (1996). The gamma distribution function is shown in equation (1), where  $k$  and  $\theta$  represent the shape and scale parameters respectively, as  $x$  represents amount of pollutants.

$$f(x; k, \theta) = x^{k-1} \frac{e^{-x/\theta}}{\theta^k \Gamma k} \text{ for } x > 0 \text{ and } k, \theta > 0 \quad (1)$$

Alternatively, the gamma distribution can be parameterized in terms of a shape parameter  $\alpha = k$  and an inverse scale parameter  $\beta = \frac{1}{\theta}$ , called a rate parameter:

$$g(x; \alpha, \beta) = \frac{\beta^\alpha}{\Gamma \alpha} x^{\alpha-1} e^{-\beta x} \text{ for } x > 0 \quad (2)$$

$$\Gamma \alpha = \int_{-\infty}^{\infty} e^{-x} x^{\alpha-1} dx \quad (3)$$

If the above gamma function is divided by  $\Gamma \alpha$ , it becomes a gamma function distribution

$$f(x) = \frac{e^{-x} x^{\alpha-1}}{\Gamma \alpha} \alpha > 0; x > 0 \quad (4)$$

This is called a one-parameter case of gamma distribution. However, for a continuous random variable  $X$  such that  $\alpha$  and  $\beta$  are fixed then the

two parameter case of gamma distribution is defined in equation (5)

$$f(x) = \frac{\beta^\alpha}{\Gamma \alpha} x^{\alpha-1} e^{-\beta x} \alpha > 0; \beta > 0; x > 0 \quad (5)$$

The  $r^{\text{th}}$  moment about the origin of equation (5) is obtained as;

$$E(x^r) = \mu_r' = \frac{\beta^r \Gamma(r + \alpha)}{\Gamma \alpha} \quad (6)$$

Setting 'r' equals 1 and 2 in equation (6) gives the mean and variance of the gamma distribution function as obtained in (7) and (8).

$$\begin{aligned} E(x) &= \mu_1' = \frac{\beta^1 \Gamma(1 + \alpha)}{\Gamma \alpha} \\ E(x) &= \frac{\alpha \beta \Gamma \alpha}{\Gamma \alpha} \\ E(x) &= \alpha \beta \end{aligned} \quad (7)$$

Similarly for  $r = 2$

$$\begin{aligned} E(x^2) &= \frac{\beta^2 \Gamma(\alpha + 2)}{\Gamma \alpha} \\ E(x^2) &= \frac{(\alpha + 1) \Gamma(\alpha + 1)}{\Gamma \alpha} \beta^2 \\ E(x^2) &= \alpha^2 \beta^2 + \alpha \beta^2 \\ S^2 &= E(x^2) - (E(x))^2 \\ S^2 &= \alpha \beta^2 \end{aligned} \quad (8)$$

For this study, the gamma distribution parameters are estimated through the maximum likelihood estimation (MLE).

Taking the natural log of likelihood function of equation (5), we obtain

$$\begin{aligned} \ln \prod_{i=1}^n f(x) &= \frac{\sum x / \beta - \sum \ln x^{\alpha-1}}{\Gamma(\alpha)^n \beta^{n\alpha}} \\ &= -\frac{\sum x}{\beta} + \ln \prod_{i=1}^n x^{\alpha-1} - \ln \Gamma(\alpha)^n - \ln \beta^{n\alpha} \\ &= -\frac{\sum x}{\beta} + (\alpha - 1) \sum \ln x - \ln \Gamma(\alpha)^n - n \alpha \ln \beta \end{aligned} \quad (9)$$

The partial derivative of (6) with respect to  $\beta$  yields its estimate as in (7);

$$\frac{\delta \ln \prod f(x)}{\delta \beta} = \frac{\sum x}{\beta^2} - \frac{n\alpha}{\beta} = 0$$

$$\Rightarrow \hat{\beta} = \frac{\bar{x}}{\hat{\alpha}} \quad (10)$$

Also, differentiating (6) partially with respect to  $\alpha$  gives the estimate in (11);

$$\hat{\alpha} = \frac{1}{4A} \left[ 1 + \sqrt{1 + \frac{4A}{3}} \right] \quad (11)$$

$$\text{Where; } A = \ln(\bar{x}) - \frac{\sum_{i=1}^n \ln(x_i)}{n} \quad (12)$$

(Husak *et al.*, 2006)

The calculation of MLEs starts with the derivation of an intermediate value 'A' as shown in equation (12) (Wilks *et al.*, 1995; Ozturk *et al.*, 1981; and Thom *et al.*, 1958), where  $x_i$  equal respective values of pollutants, and the mean ( $\bar{x}$ ) is the arithmetic mean of all pollutant values. Ultimately, 'A' is a measure of the skewness of the distribution, whose value is then used to estimate the shape parameter ' $\hat{\alpha}$ ' in (11). Interestingly, the product of the shape parameter ( $\hat{\alpha}$ ) and the square of the scale parameter ( $\hat{\beta}^2$ ) equals the variance as established in (8).

### Generalized Linear Model (GLM)

The GLM is a flexible generalization of ordinary least square regression. It generalizes linear regression by allowing the linear model to be related to the response variable via a link function, and by allowing the magnitude of the variance of each measurement to be a function of its predicted value. Cullagh *et al.* (1989).

In this paper, it has been attempted to embed the work and the procedure within the framework of a probability model. In most cases, if there are two  $x$  values that are the same, the associated  $y$  values will differ. Considering  $y_i$  as observed value

of a random variable  $Y_i$  whose distribution depends on  $x_i$ . An appropriate model for such a situation is;

$$Y_i = \beta_0 + \beta x_i + \epsilon_i$$

Where  $\epsilon_i$  are independent random errors with zero mean and common variance  $\sigma^2$ .

Then;  $E(Y_i) = \beta_0 + \beta x_i = \mu(x_i)$ , say

If  $X_i = x_i$ , and we are making extra assumption that the  $\epsilon_i$ 's are normally distributed, then predicting a probability distribution for  $Y$  given  $x$  means that;

$$Y_i \approx N(\beta_0 + \beta x_i, \sigma^2) \quad (\text{Krzanowski } et al., 1998.)$$

The problem considered here is to determine the concentration or amount of pollutants in Asejire reservoir. The quantity of interest is called the dependent or response variable and other quantities are explanatory variables, predictors, or covariates. The response variable here is Turbidity and the potential predictors are pH, Colour, Dissolved Oxygen and Alkalinity.

Using the statistical package 'STATA' we obtain the GLM for the pollution data, and the results are as summarized in table 3.

### Goodness of Fit Test

Having tried to model with gamma distribution, the need to determine the goodness of fits arises. Statistical intervals and test are often on specific distributional assumptions, before computing or carrying out any analysis on given or observed data in any area of interest, there is a need to verify or check if some assumptions are justified, otherwise, the correction has to be done on the data after pre-test to make it valid and suitable for intended analysis.

### The W-Test

This test developed by Shapiro and Wilk (1965) is an effective method for testing whether data set have been drawn from an underlying normal distribution. The null hypothesis to be tested

is

$H_0$ : The population has a normal distribution

$H_1$ : the population does not have a normal distribution.

If  $H_0$  is not rejected, then the data set is consistent with the  $H_0$  distribution, although a retest in using additional data could result in rejecting  $H_0$ .

**The Kolmogorov - Smirnov Test**

Once the parameters are estimated, their accuracy in approximating the true distribution must be confirmed. To do this, the estimated gamma distribution is compared against the empirical distribution (Ison *et al.*, 1971). This is done with Kolmogorov - Smirnov (KS) goodness of fit test (Crutcher *et al.*, 1975). The Kolmogorov - Smirnov test compares the cumulative distribution functions of the theoretical

distribution - the distribution described by the estimated shape and scale parameters - with the observed values and returns the maximum difference between these two cumulative distributions (Wilks *et al.*, 1995). This maximum difference in cumulative distribution functions is frequently referred to as the KS - Statistic.

The acceptable KS value for rejection depends on the number of points in the empirical distribution being used to test the theoretical distribution, and the rejection level chosen for the study. (Crutcher *et al.*, 1975) Because the acceptable value of the KS-statistic is a variable, the confidence in accepting or rejecting the theoretical distribution may be measured by the P-value, which incorporates the number of values used in the test into the calculation of its value. Small P-value calls for rejection of the null hypothesis, while a P-value larger than the selected significance level means the null

Table 1: Estimates of Parameters

Probability Distribution	Parameters	Turbidity	Colour	pH	Dissolved Oxygen	Alkalinity
Gamma	$\alpha$	8.6729	162.6217	199.9002	85.6105	19.3603
	$\beta$	0.3092	0.0309	0.0374	0.0887	1.9980

Table 2: Summary table for the fitted two-parameter Gamma Distribution

Mean and Variance	Pollutants				
	Turbidity	Colour	pH	Dissolved Oxygen	Alkalinity
$\alpha\beta$	2.6818	5.0333	7.4900	7.5967	38.6833
$\alpha\beta^2$	0.8297	0.1553	0.2796	0.6742	77.2941

Table 3: Generalized Linear Model Results

	Coefficients	Standard Error	P-Value	95% Confidence Interval	Remark
Col	0.12952	0.24281	0.019	-0.34637 , 0.60542	Significant
pH	0.03197	0.19134	0.026	-0.34304 , 0.40700	Significant
DO	0.08623	0.12796	0.500	-0.16457 , 0.33704	Not Significant
Alk	0.01187	0.01167	0.039	-0.01100 , 0.03474	Significant

Col = Colour. DO = Dissolved Oxvaen. Alk = Alkalinity

Table 4: Shapiro-Wilk Test Result

Variable	Obs	W	V	Z	Prob>z
Turbidity	60	0.89675	5.612	3.718	0.00010
Colour	60	0.95278	2.567	2.032	0.02108
pH	60	0.94523	2.977	2.352	0.00935
Dissolved Oxygen	60	0.85915	7.656	4.388	0.00001
Alkalinity	60	0.90214	5.319	3.603	0.00016

Obs - Observation; W - quantiles of Shapiro-Wilk test for Normality provided in the table ( $W_{\alpha} = W_{0.05} = 0.967$ ); V - index for departure from normality; Z - test of statistical significance

Table 5: Shapiro-Francia Test Result

Variables	Obs	$W_a$	V	Z	Prob>z
Turbidity	60	0.90470	5.705	3.269	0.00054
Colour	60	0.95893	2.458	1.729	0.04190
pH	60	0.95199	2.874	2.020	0.02167
Dissolved Oxygen	60	0.85551	8.650	4.003	0.00003
Alkalinity	60	0.89355	6.372	3.466	0.00026

$W_a$  - Shapiro - Wilk W test for Normality

Table 6: Kolmogorov-Smirnov Test Result

Parameters	Turbidity	Colour	pH	Dissolved Oxygen	Alkalinity
D	1.0000	1.0000	1.0000	1.0000	0.5284
P-value	0.100	0.070	0.100	0.080	0.074

D - the maximum difference between the empirical and theoretical distributions

hypothesis cannot be rejected (Wilks *et al.*, 1995).

$X \sim \text{gamma}(\alpha, \beta)$   
 $X \sim \text{gamma}(8.6729, 0.3092)$

## Results

### Parameter Estimation

Having obtained the expressions for the shape parameter " $\alpha$ " and scale parameter " $\beta$ " as in (10) and (11) above using the method of maximum likelihood estimate, the estimates of the two parameters " $\alpha$ " and " $\beta$ " for each of the water pollutants considered using statistical package "R" are as summarized in Table 1.

### Fitting Two - parameter gamma for Pollutants

From table 1 above, we observe that turbidity observation 'X' has gamma distribution with shape parameter  $\alpha = 8.6729$  and scale parameter  $\beta = 0.3092$ , statistically written as:

$$f(x; 8.6729, 0.3092) = \frac{\ell^{-x} 0.3092^x x^{8.6729-1}}{\Gamma(8.6729) 0.3092^{8.6729}} \quad (13)$$

$$E(X) = \alpha\beta = 2.6818$$

$$V(X) = \alpha\beta^2 = 0.8297$$

Also for **colour**;  $x \sim \text{gamma}(162.6217, 0.0309)$

$$f(x, 162.6217, 0.0309) = \frac{\ell^{-x} 0.0309^x x^{162.6217-1}}{\Gamma(162.6217) 0.0309^{162.6217}} \quad (14)$$

$$E(x) = \alpha\beta = (162.6217)(0.0309)$$

$$E(x) = 5.0333$$

$$V(x) = \alpha\beta^2$$

$$V(x) = (162.6217)(0.0309)^2 = 0.1553$$

Following the same procedure as above we obtain estimate of means and variances for all other pollutants, and the results are as tabulated in table 2.

**Discussion**

In table 2, the average amount of turbidity is 2.6818 mg/l which was higher than the World Health Organization (WHO) standard of less than 1 mg/l for treated water. Also, the pH of water in the reservoir was 7.49 which conforms with the WHO Standard of between 6.5 and 8.5 mg/l. The mean amount of colour in the water was 5.0333 H.U; this is above the WHO standard value of 5.0 H.U, while the Dissolved oxygen and Total Alkalinity quantity in the reservoir were 7.5967 mg/l and 38.6833 mg/l respectively, which are also in consonance with the WHO Standard of portable water quality. However, it is evident here that the reservoir was polluted considering the level of turbidity it contained. It was also observed from the generalised linear model results that the coefficient of all the variables considered are significant except that of Dissolved Oxygen (DO) with P-values greater than 0.05. This means that the dissolved oxygen is the weakest of all the considered pollutants.

Moreso, fitting the response variable against the independent variables, the model yielded R<sup>2</sup> of 0.4203. This simply means that 42% of the variation in pollution is jointly explained by all the variables (pollutants), an indication of the level of pollution in Asejire reservoir, informing that the water was fairly polluted.

The results obtained from Shapiro-Wilk and Shapiro-Francia are also discussed as shown in tables 4 and 5 above. Since we will reject hypothesis of supporting a gamma distribution at  $\alpha$  significance level if W is less than the quantile provided in table (quantiles of Shapiro-Wilk test for Normality) Values of W, such that 100P% of

the distribution of W is less than Wp on table at  $\alpha = 0.05$  i.e.  $W_{\alpha} = W_{0.05} = 0.967$

Setting

$W_a =$  Shapiro - Wilk statistics for Normality

$W_b =$  Shapiro - Francia statistic for Normality

In the above results, all pollutant values and the response variable have  $W_a$  less than 0.967 (tabulated value). This clearly indicates that the data can arise from another distribution other than normal distribution, and of course buttressing the fact that all the variables are not normally distributed. The general results also follow a strong support for the claim that environmental data are positively skewed, hence the application of the gamma distribution.

Moreover, once the gamma distribution parameters have been estimated, they should be evaluated to understand how accurately they reflect our data, and therefore represent the modelled probability of pollutants in the reservoir. The KS test offers a straightforward method for assessing the relationship between the empirical distribution and the estimated distribution, leading to either acceptance or rejection of the null hypothesis. As used here, the null hypothesis of the KS test is that the pollutants follow gamma distribution with parameters as estimated in equations (10) and (11). The critical region (acceptance or rejection region) is arbitrary, as  $\alpha = 0.05$  was used in this work, meaning that we reject the null hypothesis that the theoretical distribution is performing adequately in modelling the pollutants with p-value less than 0.05. The test produced a P-value  $> 0.05$  for all the variables; hence, we fail to reject the null hypothesis that the data follow a gamma distribution.

**Conclusion and Recommendation**

Summarily, this paper presented estimation of water pollution parameters for the gamma distribution using the maximum likelihood estimates. The ability of the gamma distribution

and parameter estimates to adequately fit the empirical distribution of values in the model was tested using the Kolmogorov-Smirnov (KS) test of goodness of fit. The test showed that the gamma distribution and the estimated parameters could not be rejected as a suitable distribution for the water pollution data at a 0.05 level of significance. The hypothesis testing here indicates that the gamma distribution provides a reasonable description and evaluation of pollutants in the reservoir. Through analysis of the distribution parameters, it is possible to examine the amount of pollutant in the reservoir and see if it is outside the World Health Organization (WHO 2011) standard for portable water or not. The ultimate application of the information presented in this paper reveals the true value of this research as we conclude that the reservoir is polluted and we recommend that;

1. Government should set up a committee, which will ensure that statistically verifiable ideal standards are maintained before the water is treated and passed out to the people for drinking.
2. Waste organic materials generated from the plant (company) which are washed into the drain should be reduced so that the Dissolve Oxygen will rise and survival of aquatic life will be better, and finally we would like to mention that;
3. Regular environmental audit on Asejire reservoir is done, so that government will ascertain the level of pollutants in the effluents as compared with the existing standards.

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